Lecture 23
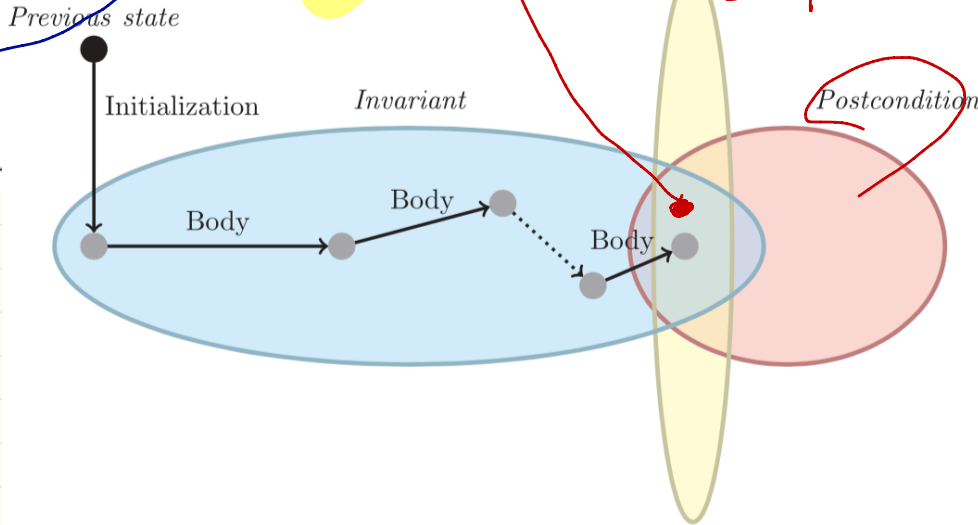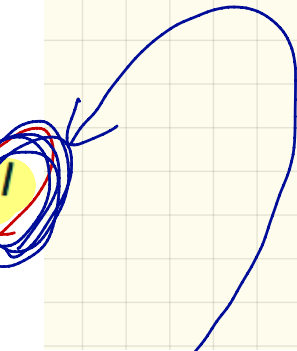## Monday Dec. 4

```
from
  S_init
invariant
  invariant_tag: I
until
  B
loop
  S_body
variant
  variant_tag: V
end
```

ensure R

end

Previous state

Initialization

Invariant

Body

Body

Body

Exit condition

Postcondition

terminating
state:
① B
② I
③ R

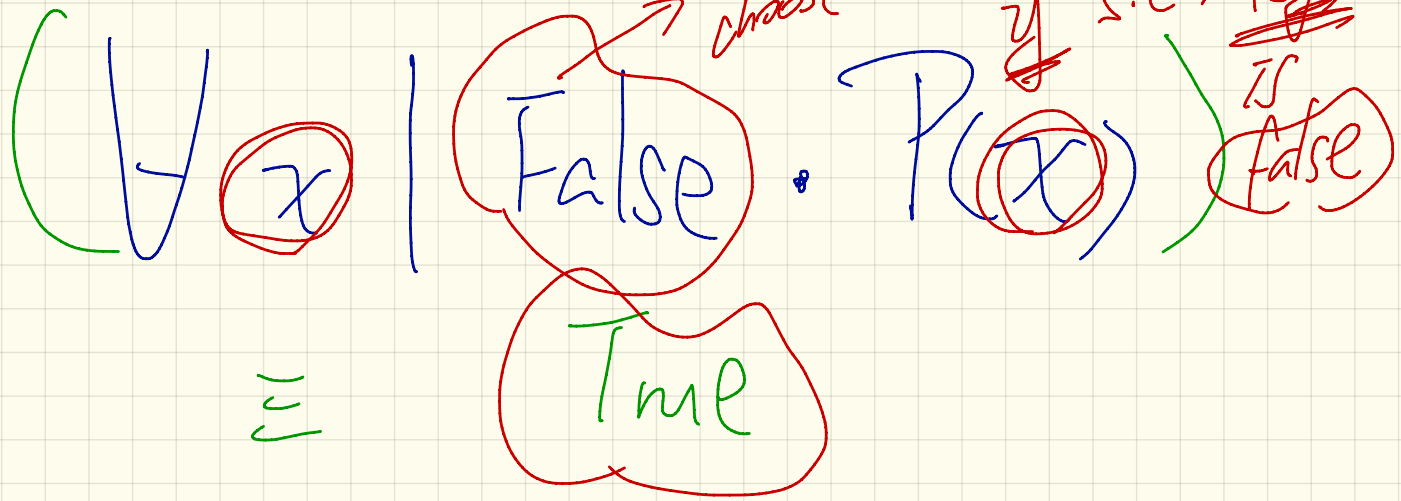Consider the feature call find_max(⟨⟨20, 10, 40, 30⟩⟩), given:

a.lower 1
a.upper 4

- **Loop Invariant**: $\forall j \mid a.lower \leq j < i \cdot Result \geq a[j]$
- **Loop Variant**: $a.upper - i + 1$
- **Postcondition**: $\forall j \mid a.lower \leq a.lower \leq a.upper \cdot Result \geq a[j]$

AFTER ITERATION	i	Result	LI	EXIT (i > a.upper)?	LV
Initialization	1	20	T	F	–
1st	2	20			3
2nd	3				2
3rd	4				1
4th	5				0

$\forall j \mid a.lower \leq j < 2 \cdot 20 \geq a[j]$ $j = 1$

$20 \geq a[j]$ N

of the end of 1st it.s

3 more to go

Init:
  $i := 1$
  Result $:= a[i]$ (20)

$\forall j \mid 1 \leq j < 1 \cdot 20 \geq a[j]$
  F = T

$$(\forall x \mid \text{False} \cdot P(x))$$

no way to choose a witness y s.t. $P(y)$ is false

$\equiv$

True

```
from
    S_init
invariant
    invariant_tag: I
until
    B
loop
    S_body
variant
    variant_tag: V
end
```

B → boolean

V → integer

As long as
soon as
B is false

⇒ more iterations

⇒ $V > 0$

⇒ V ⇒

$$\widehat{S} = \widehat{T}$$

$$S \subseteq T \wedge T \subseteq S$$

return ITERABLE
set of keys whose
assoc. values are v.

mode of dictionary
(function) with its
range restricted to 'v'